



Improving fraction learning outcomes through differentiated problem-based learning using manipulative media

Badra Intan Mei Alviani¹, Dwi Handayani², Anatri Desstya³

^{1,2,3} Universitas Muhammadiyah Surakarta, Surakarta, Indonesia

badraintan13@gmail.com¹, dwihanday06@gmail.com², ad121@ums.ac.id³

ABSTRACT

Fifth-grade students' fraction learning outcomes remain low because fractions are abstract, whereas students operate at a concrete stage. Passive lecture methods and limited instructional media exacerbate this gap. Thus, an approach aligning with students' cognitive development is needed to bridge abstract concepts with concrete experiences. This study aimed to improve learning outcomes in adding and subtracting fractions with different denominators using differentiated Problem-Based Learning (PBL) supported by Multiplication and Butterfly Boards. Following the Kemmis and McTaggart Classroom Action Research model over two cycles, the study involved 11 fifth-grade students at SD Muhammadiyah 7 Joyosuran Surakarta. The study analyzed data collected through tests, observations, and documentation quantitatively and qualitatively. Results showed the average score increased from 19.27 (pre-cycle) to 67.81 (Cycle I) and 93.73 (Cycle II). Classical mastery rose from 0% to 45.4%, reaching 100% in Cycle II. The average N-Gain score of 0.92 fell into the high category. These findings show that differentiated PBL supported by manipulative media effectively improves students' learning outcomes in fraction operations. This study implies that combining differentiated learning with manipulative media fosters active student engagement in mathematics.

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ABSTRAK

Hasil belajar pecahan siswa kelas lima masih rendah karena pecahan bersifat abstrak, sedangkan siswa masih berada pada tahap konkret. Metode ceramah pasif dan media pembelajaran yang terbatas semakin memperparah kesenjangan ini. Oleh karena itu, diperlukan pendekatan yang selaras dengan perkembangan kognitif siswa untuk menjembatani konsep abstrak dengan pengalaman konkret. Penelitian ini bertujuan untuk meningkatkan hasil belajar dalam penjumlahan dan pengurangan pecahan dengan penyebut yang berbeda menggunakan Pembelajaran Berbasis Masalah (Problem-Based Learning/PBL) yang terdiferensiasi, didukung oleh Papan Perkalian dan Papan Kupu-kupu. Dengan mengikuti model Penelitian Tindakan Kelas Kemmis dan McTaggart selama dua siklus, penelitian ini melibatkan 11 siswa kelas lima di SD Muhammadiyah 7 Joyosuran Surakarta. Data yang dikumpulkan melalui tes, observasi, dan dokumentasi dianalisis secara kuantitatif dan kualitatif. Hasil menunjukkan bahwa nilai rata-rata meningkat dari 19,27 (pra-siklus) menjadi 67,81 (Siklus I) dan 93,73 (Siklus II). Tingkat penguasaan klasik meningkat dari 0% menjadi 45,4% dan mencapai 100% pada Siklus II. Nilai rata-rata N-Gain sebesar 0,92 termasuk dalam kategori tinggi. Temuan ini menunjukkan bahwa PBL yang terdiferensiasi dan didukung oleh media manipulatif secara efektif meningkatkan hasil belajar siswa dalam operasi pecahan. Penelitian ini menyimpulkan bahwa penggabungan pembelajaran terdiferensiasi dengan media manipulatif mendorong keterlibatan aktif siswa dalam pembelajaran Matematika.

Kata Kunci: hasil belajar; media manipulatif; pecahan; pembelajaran berbasis masalah; pembelajaran berdiferensiasi

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INTRODUCTION

Mathematics plays a strategic role in elementary education because it trains students to think logically, systematically, critically, and creatively when solving everyday problems. It is not limited to computational skills but also encompasses conceptual understanding and its application in real-world situations (Nashiroh & Desstya, 2024). However, in practice, mathematics is still widely perceived as difficult, especially at the elementary school level (Afrianti et al., 2026). Adding and subtracting fractions is one of the most common challenges for fifth-grade students. Because fractions are abstract, students in the concrete operational stage of Piaget's theory of cognitive development require a concrete approach (Ijirana et al., 2024). When instruction does not provide real and meaningful experiences, students tend to memorize procedures without understanding the underlying concepts, leading to difficulties in determining the value of fractions, finding common denominators, and solving word problems involving fractions.

Preliminary observations in Class VB at SD Muhammadiyah 7 Joyosuran indicate that learning outcomes in fractions remain low; most students have not yet met the school's Learning Achievement Criteria (KKTP) of 75 (out of a maximum score of 100). Previous studies have reported similar issues, as fractions are often considered abstract and difficult for elementary school students, contributing to low mastery of mathematics learning outcomes (Putri et al., 2025). This low achievement is influenced by several factors: (1) teachers still rely on conventional lecture methods and repetitive drills without adequate learning media; (2) the learning process has not fully accommodated differences in students' abilities, interests, learning styles, and needs; and (3) students are not actively engaged in discovering and understanding the concept of fractions. The diversity of student characteristics within a single classroom presents its own challenges.

Each student has different levels of readiness, academic abilities, interests, and learning profiles, so teachers need an approach that meets all students' learning needs. Differentiated instruction adapts content, processes, and outcomes to students' needs, allowing teachers to provide more effective instruction and ensure every student has optimal learning opportunities (Azmy & Fanny, 2023). In addition to a differentiated approach, selecting the appropriate learning model is crucial for the quality of the learning process and its outcomes. Problem-Based Learning (PBL) effectively enhances critical thinking skills and learning outcomes (Suryanti et al., 2026). PBL utilizes contextual problems as the starting point for learning (Hafizah et al., 2024). Through PBL, students learn to identify problems, seek information, engage in discussion and collaboration, and independently discover solutions; this encourages them to become more active and creative while helping them connect mathematical concepts to everyday life.

When studying fractions, manipulatives are also necessary to help students turn abstract concepts into something more concrete (Yıldırım & Yıkılmış, 2022). Manipulatives allow students to see, touch, and manipulate objects directly, making fractions easier to understand; a lack of concrete learning materials is one reason students understand fractions poorly (Islamiah et al., 2025). In this study, the teaching aids used were an interactive board that students could manipulate directly to find the Least Common Multiple (LCM) and equalize denominators, as well as a visual board illustrating the steps of fraction operations using a butterfly-wing pattern. These teaching aids, called the Multiplication Board and the Butterfly Board, are intended to help students understand equivalent denominators, determine the LCM, and perform fraction addition and subtraction more easily, concretely, and meaningfully.

Various studies indicate that implementing Problem-Based Learning (PBL) combined with differentiated instruction and manipulatives can improve mathematics learning outcomes for elementary school students, particularly in fractions. One study found that concrete and virtual manipulatives are equally effective in improving understanding of fractions and motivation to learn mathematics; this suggests that integrating both yields better learning outcomes than using either method in isolation (Ukdem & Çetin, 2022). Other research also indicates that a differentiated approach within problem-based learning improves students' mathematics learning outcomes (Agusdianita et al., 2024). Furthermore, research

recommends PBL as an approach to support students' problem-solving processes within the mathematics curriculum (Usta et al., 2026).

Although PBL, differentiated instruction, and manipulatives have each been proven effective when applied separately, the combination of a PBL model integrated with a differentiated approach and supported by concrete manipulatives has not yet been specifically studied for teaching fractions at the research site; it is expected to provide a more contextual and student-centered learning experience that better accommodates diverse learning styles. This gap constitutes the uniqueness and urgency of this study. Based on this background, this classroom action research implemented differentiated Problem-Based Learning, using manipulatives such as the Multiplication Board and the Butterfly Board, with the objectives of 1) Determining whether this intervention could improve mathematics learning outcomes on the topic of fractions among students in Class VB at SD Muhammadiyah 7 Joyosuran; 2) Describing the improvement in learning outcomes from the pre-cycle to Cycle I; and 3) describing the improvement in learning outcomes from Cycle I to Cycle II following refinements to the intervention.

LITERATURE REVIEW

Mathematics Learning Outcomes in Elementary School

Mathematics learning outcomes are constructed as manifestations of multidimensional competencies that encompass students' intellectual, emotional, and motor domains (Gün & Bulut, 2023). In line with constructivist principles, learning outcomes reflect how students transform past experiences into new, meaningful structures of understanding. In the fifth grade, mathematics plays a strategic role in honing logical thinking, even though the subject's abstract nature often clashes with the cognitive limitations of students at the concrete operational stage.

Students must internalize every fundamental element of mathematics through an adaptive learning approach to avoid misconceptions (Netriwati et al., 2025). The key indicators of success for this process are as follows (Widianti et al., 2024): 1) Solve problems systematically, structuring the steps for solving problems in a logical sequence; 2) Demonstrate strong reasoning, developing logical arguments regarding mathematical operations; and 3) Communicate effectively, communicating mathematical concepts effectively, both orally and in writing. Based on this, the success of the interventions in this action research is measured by improvements in students' learning outcomes in the cognitive domain.

Fractions and Their Characteristics

Fractions are one of the most challenging topics in the fifth-grade elementary school mathematics curriculum. The main challenge of this topic lies in its level of abstraction, which is significantly higher than that of previous number-related topics. Students are required to leap from an understanding of whole, concrete natural numbers to the abstract concept of rational numbers, where a value is no longer represented by a single number but rather by a relational structure (part-whole relationship): a) Numerator (top number) indicates how many parts are taken; and b) Denominator (bottom number) indicates the total number of parts in the whole. This shift in mindset often confuses students, particularly when performing arithmetic operations, understanding place value, and comparing fractions with different denominators.

Without manipulatives, students tend to memorize formulas mechanically without understanding the basic concepts of numerators and denominators (Khoo et al., 2025). Therefore, a cognitive bridge based on concrete visualization is needed to reduce abstraction. This can be achieved through manipulatives, including fraction blocks, fraction wheels, origami paper, and concrete games such as a numeracy-based Snakes and Ladders game (Ardi & Dessty, 2023). Consequently, teachers must be more creative in

designing contextual learning activities so these materials can build students' mathematical thinking from an early age.

Integration of Differentiated PBL with Manipulative Media

The synergy between the PBL model, a differentiated learning approach, and manipulative media creates a holistic and inclusive learning framework. In this learning ecosystem, the teacher presents authentic problems that are accessible to all students based on their readiness levels and learning profiles, with the "*Butterfly Board*" manipulative serving as the primary investigative tool during the exploration phase. This integration optimally accommodates diverse learning styles (Annamalai & Sazan, 2025):

1. Kinesthetic learning style: Facilitated through physical manipulation (marking with markers on the Multiplication Board and attaching fraction cards to the Butterfly Board).
2. Visual learning style: Clearly understanding concepts through the color representation of marker dots and the shapes of fraction paths on the demonstration boards.
3. Auditory learning style: Internalizing understanding through argumentative discussions while collaborating to solve problems.

The instructional design is implemented systematically through the five main stages of the PBL syntax, which are then integrated with differentiation and the "*Butterfly Board*" medium (Ayubi et al., 2025):

1. Student problem orientation: The teacher presents a contextual problem through the hands-on activity of dividing pieces of paper. The teacher applies content differentiation: an oral narrative is presented to facilitate auditory learning styles, and images of fractions are displayed via PowerPoint to accommodate visual learning styles.
2. Organizing students for learning: Process differentiation is applied by grouping students heterogeneously based on learning style mapping and cognitive readiness (Büscher, 2021):
 - a. Tutorial group (easy level): Receives intensive assistance in using the Multiplication Board and Butterfly Board media, along with easy-level worksheets equipped with detailed scaffolding.
 - b. Developing group (intermediate level): Uses intermediate-level worksheets with step-by-step, layered guidance.
 - c. Independent group (advanced level): Utilizes a combination of the Butterfly Board and advanced-level worksheets that involve problem-solving through more complex word problems.
3. Guiding individual and group investigations: Kinesthetic-based differentiation is actively implemented as students discuss ways to visualize mathematical operations concretely until they find a solution.
4. Developing and presenting work: Product differentiation is accommodated by allowing groups to freely choose how to present their problem-solving results, such as by demonstrating them directly in front of the class.
5. Analyzing and evaluating the problem-solving process: A step involving reflection and cognitive reinforcement, in which the teacher and students evaluate the entire problem-solving process that has taken place.

METHODS

This research used Classroom Action Research (CAR) and adopted the spiral model proposed by Kemmis and McTaggart. CAR aims to improve teacher performance and enhance student learning outcomes in a specific classroom context through self-reflection (Semathong, 2023). The research was designed in two cycles, with each cycle comprising planning, action implementation, observation, and reflection. The research subjects were 11 students in Class VB at SD Muhammadiyah 7 Joyosuran during the even

semester of the 2025/2026 academic year, with the classroom teacher as the implementer of the action. The research focused on students' mathematics learning outcomes in adding and subtracting fractions with different denominators, as well as the implementation process of differentiated PBL supported by manipulative media, namely the multiplication board and the butterfly board.

The data collected in this action research included pre-cycle, Cycle I, and Cycle II scores, with a maximum score of 100. Data collection used a combination of testing, observation, and documentation. The test instrument consisted of 15 multiple-choice questions, 5 short-answer questions, and 3 essay questions administered during the pre-test, post-test I, and post-test II to measure students' cognitive abilities in adding and subtracting fractions. An observer used a checklist to record PBL implementation, teacher activities, and student engagement. Documentation included worksheets, attendance sheets, reflection journals, and photographs of learning activities.

Data validity was assessed through source triangulation, methodological triangulation, member checks with the mentor teacher/collaborator after each cycle, and enhanced observational rigor and peer discussions. Data analysis combined quantitative and qualitative approaches. Quantitative data were processed to calculate the class average using the following formula:

$$\left(x = \frac{\sum x}{N} \right)$$

Calculate the traditional completion rate using the formula:

$$\left(P = \frac{\text{jumlah siswa tuntas}}{\text{jumlah seluruh siswa}} \times 100\% \right)$$

Calculating N-Gain based on Hake's criteria using the formula:

$$\left(a = \frac{S_{post} - S_{pre}}{S_{maks} - S_{pre}} \right)$$

Qualitative data were analyzed through data reduction, data presentation, and conclusion drawing. This study aims to test the effectiveness of the intervention, with success indicators including an increase in the average score across cycles, a minimum classical mastery rate of 80% (at least 9 students achieving a KKTP score of ≥ 70), and an N-Gain score of at least the moderate category ($g > 0.3$).

RESULTS AND DISCUSSION

Results

Before implementing the intervention, the researcher conducted preliminary observations and administered a pre-test to all 11 students in Class VB at SD Muhammadiyah 7 Joyosuran. The initial observation aimed to determine the current state of mathematics learning, specifically in addition and subtraction of fractions with different denominators, before implementing a differentiated PBL model supported by the Multiplication Board and Butterfly Board manipulatives. The observation results showed that fifth-grade mathematics learning still followed a conventional approach. The teacher predominantly delivered the material through lectures without utilizing concrete learning media. As a result, students struggled to understand abstract fraction concepts, particularly when finding common denominators. Additionally, the instruction did not account for variations in each student's learning readiness, interests, or learning styles.

The lack of interactive media and a monotonous approach cause students to become passive, get bored quickly, and participate less actively in the learning process (Auzi et al., 2025). Therefore, teaching methods must change so that mathematics learning becomes not merely memorization but a meaningful reasoning process for all students. The pre-test results confirmed this issue. The class average score was

only 19.27, far below the Learning Objective Achievement Criteria (KKTP) of 70. Of the 11 students, 0 students (0.0%) met the KKTP, while 11 students (100.0%) did not meet the criteria. This situation shows that students' initial understanding of fractions remains very low and requires systematic remedial action through a contextual, differentiated approach.

Based on the results of the diagnostic assessment conducted alongside the pre-test, the data showed that the majority of students faced major difficulties in three areas: (1) determining the Least Common Multiple (LCM) of two different denominators, (2) equalizing denominators before performing arithmetic operations, and (3) solving word problems involving fractions in a logical sequence. This diagnostic data served as the foundation for designing Cycle I interventions, including the initial mapping and grouping of students for process and content differentiation. The following is a summary of the improvement in student learning outcomes from the pre-cycle stage through Cycle I to Cycle II, as presented in **Table 1**.

Table 1. Summary Table of Fifth-Grade Students' Learning Outcomes in Mathematics: Fractions

Evaluation Aspects	Pre-Cycle	Cycle I	Cycle II
Class average	19,27	67,81	93,73
Number of Students Who Met the Standards (N $\geq$ 70)	0	5	11
Percentage of Students Who Met the Standards (%)	0,0%	45,4%	100,0%
Number of Students Who Did Not Meet the Standards	11	6	0
Percentage of Students Who Did Not Meet the Standards (%)	100,0%	54,6%	0,0%

Source: Research 2026

Based on **Table 1**, scores from the pre-cycle, Cycle I, and Cycle II show continuous improvement after implementing a differentiated PBL model supported by manipulatives, the Multiplication Board, and the Butterfly Board in the mathematics material on adding and subtracting fractions. In Cycles I and II, instruction was delivered through teaching modules tailored to a differentiated approach and the PBL model framework. Learning activities began with an introductory phase involving observation of the learning process and the administration of a diagnostic assessment/pre-test to measure students' initial mathematics proficiency in Class VB.

Students showed improved learning outcomes, as shown by higher average scores and a higher percentage of students meeting the learning objectives. This action research evaluated the findings by analyzing students' cognitive learning outcomes in stages. In the pre-action (pre-cycle) phase, all 11 students (100.0%) failed to meet the Learning Objective Achievement Criteria (KKTP) or did not achieve a score of ≥ 70 , resulting in a very low class average score of 19.27. In Cycle I, the class average score was 67.81 out of 11 students. A total of 5 students (45.4%) achieved mastery, but 6 students (54.6%) still did not meet the mastery criteria. In Cycle II, student learning outcomes improved significantly: all 11 students (100.0%) achieved the KKTP proficiency score (≥ 70), and no students remained below the proficiency level, with a class average score of 93.73.

Findings from the pre-cycle through Cycle II indicate that implementing a differentiated PBL model, supported by the Multiplication Board and Butterfly Board manipulatives, can significantly enhance student engagement, conceptual understanding, and mathematics learning outcomes. The PBL model was facilitated through contextual problems closely related to students' daily lives. At the same time, the Multiplication Board helped students construct a visual understanding of finding the LCM, and the Butterfly Board facilitated the visualization of cross-multiplication of denominators. Differentiated grouping and improved scaffolding in Cycle II proved effective in overcoming students' cognitive barriers, enabling all students to achieve optimal learning outcomes.

Research Results for Cycle I

Cycle I began with a planning phase that the researchers carefully designed based on findings from the initial assessment. The researchers implemented this action plan by developing PBL-based instructional modules that integrated five key components: orienting students toward the problem, organizing students for learning, guiding individual and group investigations, developing and presenting work products, and analyzing and evaluating the problem-solving process. In addition to designing the learning sequence, the researcher prepared concrete manipulatives, including a Multiplication Board to help students visually identify the LCM and a Butterfly Board to visualize cross-multiplication for equalizing fraction denominators. The “*Multiplication Board*” and “*Butterfly Board*” manipulatives are shown in **Figure 1** below:

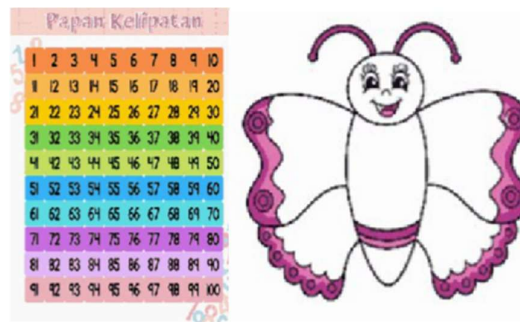


Figure 1. Manipulative Media
Source: Author's Documentation, 2026

The researcher also developed differentiated Student Worksheets (LKPD) divided into three difficulty levels: Basic, Intermediate, and Advanced, where initial student grouping was based on a relative comparison of pretest scores. Cycle I activities were conducted in a single session with a time allocation of 2 x 35 minutes, as shown in **Figure 2**.



Figure 2. Implementation of Cycle I Instruction
Source: Author's Documentation, 2026

During the introductory activity, the teacher conducted an apperception exercise using contextual prompt questions about adding the weights of sugar and flour; most students could not answer these questions correctly, which underscored the importance of using concrete materials. The lesson then moved on to the core activity by applying the entire PBL sequence in order. The teacher presented a real-life contextual problem, divided students into small groups based on ability level, and distributed differentiated worksheets. Students actively explored the Multiplication Board and the Butterfly Board to solve the problems, with the teacher providing facilitative guidance.

The activity continued with group representatives presenting their discussion results to the class, followed by a closing session of evaluation and collective reflection on the problem-solving process. Based on observations during the Cycle I learning process, the activity generally went quite well, although the observer noted two main challenges. First, student grouping was not yet optimal because it was based solely on pre-test score ranges, all of which fell on the low end below the KKTP benchmark. This created an overly wide range of ability levels among group members, so the differentiated worksheets were not yet effective in addressing each individual's needs. Second, most students still struggled with prerequisite subtopics, particularly in ordering fractions and simplifying final fraction forms.

Nevertheless, the post-test results for Cycle I showed a fairly significant improvement in learning outcomes compared to the pre-cycle conditions. The class average score increased from 19.27 to 67.81 (an increase of 48.54 points), with the number of students achieving proficiency (KKTP ≥ 70) rising from 0 students (0.0%) to 5 students (45.4%). In contrast, the remaining 6 students (54.6%) had not yet achieved proficiency. Since this achievement did not meet the established classical success indicator of at least 80%, the researcher and collaborating teachers conducted an in-depth reflection. Based on this reflection, the researchers agreed on two major improvements for Cycle II: completely overhauling the differentiation grouping based on Cycle I post-test scores to ensure greater precision, and explicitly allocating time to reinforce prerequisite material through concrete examples and guided practice during the introductory activities.

Research Results for Cycle II

Cycle II was carefully designed by integrating all reflections and improvement notes from Cycle I. The researchers, together with the collaborating teachers, restructured the learning groups based on a mapping of Post-Test I scores so the group assignments would be more targeted and aligned with students' learning readiness. The teaching module revised the time allocation for the introductory activities by adding 10 minutes specifically to review prerequisite concepts, namely ordering and simplifying fractions. In addition, the differentiated Student Worksheets (LKPD) were updated by including practice problems that explicitly reinforce these two prerequisite skills. Figure 3 shows the implementation of Cycle II instruction based on the documentation.



Figure 3. Implementation of Cycle II Instruction
Source: Author's Documentation, 2026

The Cycle II learning activities followed the revised teaching modules. During the introductory activities, the teacher reinforced prerequisite material through visual explanations and interactive question-and-answer sessions at the blackboard, which received a positive response and clarified students' confusion. In the core activities, implementing the new group-based PBL model proved far more effective. Group discussions were dynamic because group members' abilities were relatively equal, so each student faced challenges tailored to their zone of proximal development (Damanik et al., 2025). Learning also became

more engaging with a peer-tutoring strategy, in which students from the Proficient Group actively helped their peers simplify fractions using the Butterfly Board.

Observations throughout Cycle II documented a very noticeable improvement in the quality of learning. Reinforcing prerequisite concepts proved effective in eliminating students' doubts about simplifying fractions to their final form. Obstacles such as passive students or those not receiving adequate guidance were no longer found in the classroom. The learning atmosphere became more interactive, student participation in discussions and presentations increased across the board, and the "Multiples Board" media ran smoothly without technical issues. This rapid improvement in process quality directly correlates with students' cognitive learning outcomes on the Cycle II post-test.

The average class score rose sharply from 67.81 in Cycle I to 93.73 in Cycle II, representing an increase of 25.92 points. The class-wide learning completion rate reached 100.0%, with all 11 students successfully exceeding the Learning Objective Achievement Criteria (KKTP ≥ 70). Based on a reflective analysis at the end of Cycle II, all indicators of the classroom action research's success were met and exceeded the established class-wide target of at least 80%. This success stemmed from two key improvements: adjusting targeted differentiated grouping and explicitly reinforcing prerequisite material at the start of instruction. The spiral cycle of classroom action research proved effective as a mechanism for evidence-based learning improvement (evidence-based practice); therefore, the intervention was deemed successful, and the research was officially concluded at Cycle II.

Discussion

The findings indicate that improved learning outcomes resulted not merely from implementing the intervention overall, but from specific mechanisms at work across three components: the PBL model, differentiated instruction, and manipulative materials. The following section explains how each component contributes to these changes.

A Multiplication Chart Reduces the Cognitive Load Involved in Finding the LCM

Students' difficulty in determining the Least Common Multiple (LCM) largely stems from cognitive load: they must store and compare several number sequences simultaneously in working memory, which has limited capacity, especially when done mentally or algorithmically (Oral, 2021). The Multiplication Board overcomes this obstacle by shifting the comparison process from working memory to an external visual representation; students no longer need to memorize or visualize sequences of multiples, but instead look for the first point where the two colors meet. This strategy aligns with findings that visual media and concrete representations are among the most effective interventions for managing cognitive load in mathematics learning (Kuzu et al., 2025), as it reduces extrinsic load that is non-essential for concept formation (Rahmawati et al., 2024). In other words, this medium does not teach new concepts; instead, it shifts the cognitive task from memorization and calculation to visual pattern recognition, which is naturally easier for elementary school students at the concrete operational stage to process (Nabila et al., 2026). This explains why the first obstacle identified in the diagnostic assessment results (difficulty determining the LCM) was consistently overcome in both cycles.

Butterfly Board Prevents Procedural Memorization

A common mistake when equating fraction denominators is that students memorize the cross-multiplication sequence without understanding the relationships between the elements, so they easily forget or mix up the order. The Butterfly Board addresses this issue by assigning each element of the

operation (numerator, denominator, product) a fixed, meaningful spatial position; the left and right wings, antennae, and tail are not merely decorations, but a visual map of the mathematical relationship itself. Because each number's position reflects its role in the operation, students build a relational schema (why the left numerator is multiplied by the right denominator) rather than a rote rule (a sequence of steps to memorize). This structure, in line with the enactive-iconic-symbolic stages in Bruner's theory, enables students to continue solving problems correctly even when the form of the fraction or the numbers differs from the examples, an indication that the understanding formed is conceptual and relational, rather than merely a memorized procedure prone to misinterpretations of fractional representations (Chaari et al., 2026; Karika & Csíkos, 2022).

Differentiation Accelerates the Equitable Achievement of Learning Outcomes

Differentiated instruction is effective not because it provides a greater variety of tasks, but because it aligns the level of challenge with students' zone of proximal development, that is, the gap between what students can already do on their own and what they can achieve with assistance (Sanjarani et al., 2026). When the challenge is too high for less proficient students, the extra cognitive load from procedural confusion hinders conceptual understanding; conversely, a challenge that is too low for proficient students does not stimulate meaningful development. By providing scaffolding proportionate to each group, this study avoids both risks, allowing all students to reinforce concepts at the same time rather than only a subset.

The accelerated achievement of mastery in the second cycle was not merely due to the repetition of actions, but rather to the reflection from Cycle I, which identified a more specific root cause: weaknesses in prerequisite skills (ordering and simplifying fractions) that had long been a hidden barrier behind students' errors in equating denominators. By explicitly addressing this prerequisite gap before students move on to the core stage, the cognitive load previously divided between understanding new concepts and recalling basic concepts can be reduced, allowing students to focus more fully on understanding the concepts being taught (Clara et al., 2025).

The three components above complement one another because each addresses a different type of barrier: PBL provides a rationale (why students need to learn this concept, differentiation ensures that challenges match students' capacities (how much support is needed), and manipulatives provide a means of concrete representation (how abstract concepts can be processed) (Alfidyah, 2025; Herwina, 2021; Nahdhiah & Suciptaningsih, 2024; Nurcahyani et al., 2025). Without manipulatives, differentiation merely adjusts the difficulty level of problems without changing how students process abstract concepts; without differentiation, the same manipulatives would be too easy for some students and still too difficult for others; and without the PBL framework, the use of manipulatives risks becoming a mechanical activity without an understanding of why each step is taken. The integration of these three elements explains why learning outcomes improved comprehensively (100% mastery), not merely through higher average scores.

CONCLUSION

This study demonstrates that differentiated PBL using the Multiplication Board and Butterfly Board manipulatives effectively improves learning outcomes in mathematics, specifically the topic of fractions, as shown by the fulfillment of all success indicators. The sharp improvement from the pre-cycle to Cycle I demonstrates the rapid impact of the problem-based approach and concrete manipulatives, while the even greater surge in achievement in Cycle II proves that refining the intervention based on data reflection by closing gaps in prerequisite skills and improving the accuracy of grouping is the determining factor in the success of action research, not merely the repetition of actions. These findings confirm that the effectiveness of differentiated PBL supported by manipulatives depends on integrating three conditions:

the relevance of contextual problems, the appropriateness of support tailored to students' needs, and the availability of concrete representations for abstract concepts.

The study addressed challenges related to initial group assignments and doubts about prerequisite skills through data-driven group reorganization based on Post-Test I results, peer-tutoring strategies, and explicit reinforcement of prerequisite material at the start of the learning process. The main challenges encountered during the study were imprecise initial student grouping and students' uncertainty about prerequisite skills (ordering and simplifying fractions). Effective solutions to overcome these obstacles include reorganizing group assignments based on Post-Test I scores, optimizing peer-tutoring strategies from the proficient group, and explicitly allocating time to reinforce prerequisite material at the start of the learning process.

AUTHOR'S NOTE

The authors declare that there are no conflicts of interest regarding the publication of this article. The authors also affirm that the data and content of this article are free from plagiarism. The authors would like to thank the principal, the supervising teacher, and the students of Class VB at SD Muhammadiyah 7 Joyosuran for their cooperation and support during the implementation of this classroom action research.

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